The order slot and Laplace noise: two trading stages for *Competition Creates Competition*

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This note sets two trading stages for the model of *Competition Creates Competition* side by side. In the order slot, the trading stage of the paper, an informed trader and a liquidity trader are alternative arrivals in one public order opportunity. In the Laplace set-up, the informed trader chooses a continuous order and Laplace noise hides it; this is the trading stage of the manuscript of 22 October 2026 with the same title. The auction, the preparation cost and the decision of the potential rival bidder are the same in both. The Laplace manuscript calls the rival the challenger and the informed trader the investor. Section 1 states the two set-ups, Section 2 sets out what each delivers, result by result, and Section 3 names the features of each trading stage that the differences come from.

Propositions 1 to 4, Corollaries 1 to 3, Lemmas A.1 to A.5, Propositions A.1 to A.13, Corollary A.2 and the tables are those of the paper. A number with the prefix L refers to the Laplace manuscript: Proposition L-2 is its Proposition 2, and Proposition L-A.5 its Proposition A.5. Strengths that belong to the Laplace set-up carry the superscript Lap, and the Laplace noise scale is ξ.

1 The two set-ups

Shared primitives. One bidder, the incumbent bidder, or the incumbent, is already prepared. A second bidder, the potential rival bidder, or the rival, has a class $j \in \{H, L\}$, with prior $\pi = \Pr(j = H)$, and a value $\theta \sim G_j$. The classes satisfy the class ranking: G_H dominates G_L, and both supports lie above the reserve p. The incumbent's value is $R \sim F$. The rival's preparation cost is zero with probability ρ and c_H otherwise, and prepared bidders bid their values in a second-price auction with reserve p. The auction gives the proceeds t_0 without entry and t_j after entry in class j, the gross profits g_j, the target-payoff spread $\Delta_T = t_H - t_L$, gross profit $B(\mu) = g_L + \mu(g_H - g_L)$ at the belief μ, and the preparation threshold $\tau = (c_H - g_L)/(g_H - g_L)$. Entry E is the probability that the rival prepares, and high-class ownership O_H the probability that a class-H rival acquires the target.

In the example the classes are point masses at $h = 10$ and $\ell = 1$, the incumbent is uniform on $[0, r]$, and $(p, \rho, c_H, k) = (1/2, 1/4, 6, 1/50)$, with prior $1/2$ and the strengths $r_0 = 6/5$, $r_1 = 3$ and $r_2 = 18/5$. Both set-ups use this example.

The order slot. One public order opportunity comes before the rival's decision. With probability $\lambda \in (0, 1)$ the informed trader fills it: it sees the class and chooses $q \in \{-1, 0, 1\}$, and a nonzero

Table 1. The two set-ups.

	Order slot	Laplace
Order menu	$q \in \{-1, 0, 1\}$, cost k per nonzero order	$q \in [-1, 1]$, cost $k\|q\|$
What hides the order	a liquidity trader fills the slot with probability $1-\lambda$ and orders $(\beta, 1-2\beta, \beta)$	additive Laplace noise of scale $\xi > 1$
What market makers see	the order $X \in \{-1, 0, 1\}$	the flow, a real number
Largest likelihood ratio	$1+u$, $u = \lambda/[(1-\lambda)\beta]$	$e^{2/\xi}$
Posterior bounds	$[m(\pi), M(\pi)]$ after every nonzero order, and after the zero order when $\beta \le 1/3$	$[m(\pi), M(\pi)]$ at every flow
Price and belief	equal prices give equal posteriors when $\Delta_T > 0$	the rival inverts the price through the entry floor when $\Delta_T > 0$
Informed trader's problem	one comparison per class: the gross return K_j against k	a derivative bound on order sizes in $[0, 1]$
Trading bound in Proposition 2	$k < \rho\kappa(\pi)\Delta_T(F_1)$	$k < (1-1/\xi)\rho\kappa(\pi)\Delta_T(F_1)$
Primitives it adds	a unit menu, a presence probability, an exogenous liquidity motive; with research, a quadratic cost and a cap	a continuous order and a Laplace distribution with $\xi > 1$; Proposition L-A.5 extends to every density with a bounded score
Tie rules	an indifferent informed trader may mix over its best orders, and an indifferent high-cost rival prepares; the other two rules are stated where they change a conclusion (Table A.3)	an indifferent high-cost rival prepares; Propositions L-2 and L-3 need no tie rule for the informed trader

order costs k. With probability $1-\lambda$ a liquidity trader fills it and sells, abstains or buys one unit with probabilities β, $1-2\beta$ and β, where $0 < \beta \le 1/3$, for reasons apart from the sale. No one observes who filled the slot. Competitive market makers see the order $X \in \{-1, 0, 1\}$ and set $P = \mathbb{E}[V_T \mid X]$, where V_T is the terminal payoff of a target share. The device is the alternative-arrival structure of **?**, in which an informed order and a liquidity order of the same size are alternative arrivals in one opportunity; the unit menu, the presence probability and the unit cost follow **?**. The liquidity mass in one direction is $b = (1-\lambda)\beta$, and $1+u$, with $u = \lambda/b$, is the largest likelihood ratio that one order can carry. In the example $1+u = e$ and $\beta = 0.294655$, so $\lambda = 0.336122$ and $b = 0.195615$. Section 5 of the paper adds a research stage in which the informed trader chooses λ at a cost.

The Laplace set-up. The informed trader observes the class and submits an order $q \in [-1, 1]$ at the cost $k|q|$. Noise demand has the Laplace density $e^{-|z|/\xi}/(2\xi)$ with scale $\xi > 1$. Market makers see the flow, the sum of the order and the noise, and set its price at the expected terminal payoff. The likelihood ratio of a flow lies in $[e^{-2/\xi}, e^{2/\xi}]$. In the example $\xi = 2$, so $e^{2/\xi} = e$, the slot's likelihood-ratio bound.

With $1+u = e^{2/\xi}$ the two set-ups have the same posterior bounds, $M(\pi) = \pi(1+u)/[\pi(1+u) + 1-\pi]$ and $m(\pi) = \pi/[\pi + (1-\pi)(1+u)]$, and in the example $M(1/2) = e/(1+e) = 0.731059$ and $m(1/2) = 0.268941$ in both. Every order or flow has positive probability or density in both classes, so every price is on path and neither set-up needs an off-path belief. The zero-cost rival prepares after every price, so entry is at least ρ, and the gross return of a correct unit order lies between $\rho\kappa(\pi)\Delta_T$ and Δ_T, with $\kappa(\pi) = \min\{m(\pi), 1-M(\pi)\}$, in both (Lemma A.3; equation L-11). The

informed trader's problem differs: in the slot each class compares one gross return with k, while in the Laplace set-up a score bound, $|f'| \le f/\xi$ for the noise density f, shows that a correctly signed order of size s has marginal profit at least $(1 - s/\xi)$ times its gross return minus k. The slot's trading bound therefore carries no factor $1 - 1/\xi$.

2 Results side by side

Table 2 lists each result in the two set-ups. The paragraphs after it take the rows in turn.

Table 2. Results in the two set-ups.

Result	Order slot	Laplace
Beliefs and prices	Lemma A.1: $m(\pi) \le \mu \le M(\pi)$ after every nonzero order for every $\beta < 1/2$, and after the zero order when $\beta \le 1/3$. Lemma A.2: entry at least ρ; equal prices give equal posteriors. Lemma A.3: one comparison per class. Lemma A.4: orders, posteriors, entry and research depend on the auction only through Δ_T and τ.	Proposition L-A.1: the same bounds at every flow, from the score bound. Proposition L-A.2: the price reveals the posterior when $\Delta_T > 0$. Residual bounds (L-11).
Proposition 1	A stronger incumbent weakly lowers g_H and g_L for every pair of classes and weakly raises Δ_T under the class ranking, for every continuous incumbent distribution.	Proposition L-1: the same result and proof.
Proposition 2	Unique outcomes under all four tie rules, for every $\beta < 1/2$: an uninformative market at F_0, full orders with entry above ρ at F_1, full orders with entry ρ at F_2. Conditions (H1)–(H3). Ownership rises exactly when $e_H W_H(F_1) > \rho W_H(F_0)$; a condition for class H's takeover share; a decisive advantage gives outbidding.	Proposition L-2: the same three outcomes, unique against arbitrary mixed orders on $[-1, 1]$. Conditions (A1), (A2) with the factor $1 - 1/\xi$, and $B_{F_2}(M) < c_H$.
The example's region	Corollary 1 and Table 2: (H1)–(H3) hold exactly when $82/59 < u < 2686/1559$, that is $\tau(r_1) < M(1/2) < \tau(r_2)$; with $1 + u = \mathrm{e}$, exactly for priors in $(0.467850, 0.500424)$, and in $(0.467850, 0.633117)$ for parts (i) and (ii).	Appendix L-A.4: a nonempty open set in (r_0, r_1, c_H, k); priors in $(0.467850, 0.500424)$, and $(0.467850, 0.538096)$ for parts (i) and (ii); costs in $(5.997311, 6.217155)$.
Fixed information	Proposition A.2: entry is nonincreasing for every experiment and every independent cost distribution. Full orders held fixed: entry 0.522757 at every strength up to r_C, and ρ above it.	Proposition L-A.3: entry is nonincreasing. Full orders held fixed: entry falls strictly from 0.562178 at r_0 to 0.522757 at r_1.
Proposition 3	An uninformative and an informative market coexist exactly when $\pi < \tau \le M(\pi)$ and $k/(1 - \tau) \le \Delta_T \le k/(\rho \max\{\pi, 1 - \pi\})$; the informative market is then unique, and its entry rises with Δ_T while class H mixes. Along a path the strengths with coexistence form an interval when $\Delta_T(g_H - c_H)$ does not fall. Corollary 2: the whole equilibrium set on $[r_0, r_2]$; coexistence exactly on $[r_A, r_N]$. Corollary A.2: every prior.	Propositions L-3 and L-A.4: an uninformative market is an equilibrium exactly for $r \le r_N^{\mathrm{Lap}}$, and the unique one below ≈ 1.220998; full orders are unique above r_U^{Lap}. At 1.55, 1.60 and 1.65, an equilibrium with a full purchase and a partial sale coexists with the uninformative market.

Result	Order slot	Laplace
Proposition 4	Research at the prior $1/2$: the recruiting equilibrium exists exactly when $\Delta_T > 2k$ and the top posterior at the chosen research reaches τ; research and entry rise with Δ_T up to the cap. Corollary 3: the example's three outcomes and no other equilibrium. Proposition A.1: the research parameter region.	No research stage.
Robustness	Proposition A.3: cost distributions with or without atoms. Proposition A.4: every ranked pair of classes and every prior. Proposition A.5: many classes under separation. Proposition A.6: complementary signals.	Proposition L-A.6: atomless costs. Proposition L-A.5: every ranked pair. Online Appendix L-A.13: many classes. Proposition L-A.7: complementary signals.
Welfare	Proposition A.7: price access raises proceeds by 0.257948 and net surplus by 0.081960 at r_1. Proposition A.8: the payment rule. Proposition A.9 and Table 5: reserves.	Proposition L-A.9: +0.257809 and +0.080218 at r_1. Proposition L-A.8: the payment rule. Table L-5: reserves.
Empirical predictions	Section 8, Propositions A.10 to A.13: entry and incumbent strength; the price gap and its entry component; the informed share; winners; announcement returns.	Section L-7: three predictions on entry, winners and floor entry, for the net demand of the decision interval.

Beliefs and prices. The posterior bounds come from the liquidity trader in the slot and from the score bound of the noise in the Laplace set-up. In the slot the bound after a nonzero order holds for every $\beta < 1/2$, and the bound after the zero order uses $\beta \leq 1/3$. Both set-ups read the posterior from the price: in the slot, two orders with one price have the same entry and therefore the same posterior (Lemma A.2); in the Laplace set-up, the rival inverts the identity $P = t_0 + e(P)[t_L - t_0 + \Delta_T \mu]$ through the entry floor (Proposition L-A.2). When $\Delta_T > 0$ and $g_H > g_L$, the auction enters the slot's orders, posteriors, entry and research only through Δ_T and τ (Lemma A.4), and a stronger incumbent never narrows the spread and, while $g_L \leq c_H$, never lowers the threshold (Lemma A.5).

Proposition 1. The result concerns the auction alone, so it is the same in both set-ups: a stronger incumbent moves the class value gap from the rival's profit to target proceeds.

Proposition 2. The three economies have the same outcomes in both set-ups: an uninformative market against the weak incumbent, full orders that recruit after a purchase against the strong incumbent, and full orders with entry at its floor against the very strong incumbent. In the example, entry is 0.250000, 0.522757 and 0.250000 at r_0, r_1 and r_2 in both. High-class ownership is 0.125000, 0.324401 and 0.125000 in the slot, and 0.125000, 0.324192 and 0.125000 in the Laplace set-up. Uniqueness in the Laplace set-up covers mixed orders on a continuum of sizes and every unilateral deviation in $[-1, 1]$; in the slot it covers mixtures over three orders and holds under all four tie rules. In the slot, Proposition 2 also gives the class entry rates $e_H = \rho + (1 - \rho)(b + \lambda)$ and $e_L = \rho + (1 - \rho)b$, the exact ownership and takeover-share conditions, and outbidding from a decisive advantage of class H; in the example class H's takeover share rises from 0.545455 to 0.830691.

With the same posterior bounds, every margin that involves only the auction and the top

posterior is the same number in the two set-ups: the cost window $(5.997311, 6.217155)$, the prior window $(0.467850, 0.500424)$ for all three parts and the entry ceiling $r_C \approx 3.592659$, at which $\tau(r_C) = M(1/2)$. The prior window for parts (i) and (ii) reaches 0.633117 in the slot and 0.538096 in the Laplace set-up. In the slot the region in the example is an exact band for u (Corollary 1).

Fixed information. Both set-ups hold the information in the price fixed and let the rival decide again: a stronger incumbent then lowers gross profit at every belief, so entry cannot rise (Proposition A.2; Proposition L-A.3). In the slot, Proposition A.2 covers every experiment whose signal is independent of both values given the class, and every cost distribution independent of the class, the values and the signal. With full orders held fixed, the slot's frozen posteriors are three atoms $m < 1/2 < M$, and $\tau(r)$ exceeds the prior at every strength, so only the top atom recruits and entry is a step function of strength: 0.522757 up to r_C and ρ above it. In the Laplace set-up the flow at which the posterior reaches τ moves continuously with strength, so the probability of crossing it has a density, and entry falls strictly, from 0.562178 to 0.522757.

Proposition 3. In the slot, Proposition 3 gives coexistence of an uninformative and an informative market at every incumbent distribution, in terms of the two numbers Δ_T and τ. In the example at the prior $1/2$, Corollary 2 lists every equilibrium on $[r_0, r_2]$ for every slot in the band of Corollary 1. Coexistence holds exactly on $[r_A, r_N] \approx [1.393958, 1.747878]$. In the informative market class H buys at the rate $b[\Delta_T(r)/k - 2]$ up to $r_F \approx 1.467127$, so entry rises strictly from 0.454185 at r_A through 0.506112 at 1.45 to 0.522757 at r_F, and keeps that value up to r_C. The low class abstains up to r_N, mixes on (r_N, r_S) and sells for sure from $r_S \approx 2.124148$. When an indifferent high-cost rival may prepare with any probability, the threshold pair is also an equilibrium on $(r_A, r_J]$, with $r_J \approx 1.966097$, and its entry falls strictly from 0.454185 to ρ (Table A.3). At other priors (Corollary A.2), coexistence starts at r_A for priors between ≈ 0.396175 and ≈ 0.640739, starts where the prior reaches the threshold for priors up to ≈ 0.650059, and is empty below ≈ 0.396175 and from ≈ 0.650059 on.

In the Laplace set-up an uninformative market is an equilibrium exactly for $r \leq r_N^{\text{Lap}} \approx 1.747878$, the slot's r_N: against a price that carries no information, an order of size s earns $s(\rho\Delta_T/2 - k)$ at the prior $1/2$ in both set-ups. The uninformative market is the unique equilibrium below ≈ 1.220998, where $\Delta_T = k$, and full orders are unique above $r_U^{\text{Lap}} \approx 2.837417$ (Proposition L-A.4). Between these strengths, Proposition L-3 gives, at 1.55, 1.60 and 1.65, an equilibrium in which class H buys one unit and class L sells 0.460316, 0.707475 and 0.903332 of a unit, each beside the uninformative market. Entry in these equilibria rises strictly, from 0.545053 through 0.548756 to 0.551361. In the slot, at the same three strengths, entry in the informative market is 0.522757 beside the uninformative market.

Proposition 4. Only the slot has a research stage. Before it learns the class, the informed trader chooses an intensity $\lambda \in [0, \bar{\lambda}]$ at the cost $(a/2)\lambda^2$, and the intensity is the presence probability. At the prior $1/2$, when an indifferent informed trader may mix and an indifferent high-cost rival prepares, recruiting research solves $2a\lambda = \Delta_T m_\lambda - k + (\rho\Delta_T m_\lambda - k)^+$, capped at $\bar{\lambda}$, and research and entry rise with the spread up to the cap (Proposition 4). In the example, with $a = 1/40$ and $\bar{\lambda} \approx 0.336122$ (Corollary 3), zero research is an equilibrium exactly on $[r_0, r_N]$. Recruiting research is an equilibrium exactly on $[r_R, r_C]$, with $r_R \approx 1.501023$: below the cap up to $r_{\bar{\lambda}} \approx 1.677633$

and at the cap from there, with entry rising from 0.500846 to 0.522757. Floor-entry research is an equilibrium exactly on (r_N, r_Q), with $r_Q \approx 2.166995$, and on $(r_C, r_2]$, and there is no other equilibrium. At 1.55, 1.60 and 1.65, entry with recruiting research is 0.507630, 0.513965 and 0.519763. Proposition A.1 gives the cap strip $(0.290539, 0.336721)$ and, at the example's cap, the cost window $(0.021469, 0.036763)$ on which this pattern holds at the example's strengths.

Robustness. The cost-distribution results keep parts (i) and (ii) of Proposition 2 in both set-ups. In the slot the low and the high cost components can have atoms (Proposition A.3); in the example every half-width of a uniform high component below $c_H - B_{F_2}(M) \approx 0.002689$ keeps all three parts, and at half-width $1/10$ entry is 0.522757 at r_1 and 0.382711 at r_2. With atomless costs of half-width $1/10$, Laplace entry at r_1 is 0.522715 (Proposition L-A.6). For value classes, both set-ups give a nonempty open set of strengths and costs for every ranked pair. The slot's Proposition A.4 holds at every reserve, prior, zero-cost probability and slot, and Table A.4 gives six economies with their own costs and strengths; in Table L-4 of the Laplace manuscript, cases A to C meet (A1), and case D has margin -0.018122 in (A1). With many classes, separation splits the classes into a high and a low group in both set-ups (Proposition A.5; Online Appendix L-A.13). In the slot, without separation, a strict three-class equilibrium has two orders with different posterior vectors and one price. With complementary private signals, the signal example raises entry from 0.850000 to 0.879797 in the slot (Proposition A.6) and from 0.850000 to 0.879438 in the Laplace set-up (Proposition L-A.7). In the slot's example, the accuracies $(0.94, 0.505)$ raise entry from 0.250000 to 0.386933.

Welfare. Price access raises expected proceeds and acquisition surplus net of preparation costs at r_1 in both set-ups (Proposition A.7; Proposition L-A.9), and in the slot the gains are the same with research at the cap. If an order unit is a given number of shares per share of float, initial shareholders gain in total whenever that number is below 6.848394 in the slot; in the Laplace example the corresponding scale is 1.831923. The payment rule moves the spread with the sign of $1 - 2\omega$, where ω is the seller's bargaining weight, in both (Proposition A.8; Proposition L-A.8); in the slot the move is weak in general and strict when $\mathbb{E}[(R - \ell)_+]$ changes strictly and $\omega \neq 1/2$. The reserve $101/100$ excludes the low class. In the Laplace example, preparation at this reserve falls from 0.543573 at r_0 to 0.513373 at r_1, with unique continuations at both (Table L-5). In the slot, full orders are unique at both strengths and only a purchase recruits, so entry is 0.522757 at both (Proposition A.9(a)). Strict deterrence holds at the reserve $31/20$, which also excludes the weak incumbent: entry falls from 0.522757 to 0.250000 (Proposition A.9(b)).

Empirical predictions. Both set-ups give three predictions. When the price precedes the decision, a stronger incumbent can raise the probability of further preparation; when it does not, it can only lower it; and against a strong enough incumbent, preparation returns to its floor while the price stays informative. The slot derives observables from one order opportunity. Under full two-sided trade at a prior below the threshold, the purchase impact weighted by the frequency of a purchase exceeds the sale impact weighted by the frequency of a sale exactly when a purchase recruits (Proposition A.10). The price gap $P(1) - P(-1)$ is 0.786299 at r_1, and 90.2% of it is the entry component. Order counts identify the informed trader's presence at a known prior away from $1/2$, and the sale impact identifies it at any known prior; at the prior $1/2$ the shares consistent with counts and incentives at r_1 lie between ≈ 0.250299 and 0.760000 (Proposition A.11). Under full two-sided trade a prepared

rival wins more often after a purchase than after a sale exactly when, with positive probability, a high-class value beats the incumbent more often than a low-class value; this holds also above r_C, where orders do not predict entry (Proposition A.12), and the mean price change over preparation reports is positive exactly when preparation is still uncertain after some order (Proposition A.13). In the Laplace set-up the price is that of the net demand over the decision interval, and its three predictions are stated for that price.

3 What separates the two set-ups

The order menu. A continuous menu gives the low class a sale of any size. Its marginal profit falls strictly with the size of the sale, so a root of its first-order condition is a global optimum (Online Appendix L-B.2), and the partial sales of Proposition L-3 follow. On the unit menu a sale is all or nothing, and the continuous margin is a mixture probability: the low class mixes on (r_N, r_S) when an indifferent informed trader may mix (Corollary 2; Table A.3). The same holds for the rise of entry at intermediate strengths. In the Laplace set-up the size of the sale carries it; in the slot, the high class's purchase rate carries it on $[r_A, r_F]$ at a fixed presence probability, and the research intensity carries it on $[r_R, r_{\bar{\lambda}}]$. The slot's rise at a fixed presence probability lies below the three Laplace strengths, and its rise with research passes through them at lower entry.

The number of prices. The slot has three prices, so entry moves in steps along the path, and a strict fall at fixed orders needs the threshold to cross one of the three posteriors. Hence entry with full orders held fixed is flat up to r_C, and at the reserve that excludes the low class, strict deterrence needs the stronger incumbent to push gross profit at the top posterior below c_H (Proposition A.9(b)). The Laplace flow has a continuum of prices, the crossing probability has a density, and both falls are strict. In return, the three prices make the equilibrium set along the path a finite list: each class has three orders, and each gross return falls strictly with the class's own rate, so Corollaries 2 and 3 list every equilibrium at every strength from r_0 to r_2. In the example, full two-sided trade is the unique outcome for $r_S < r < h$ (Section 8).

What the market observes. Both set-ups price one trade before the rival's decision. The slot is one public order opportunity at a known date, and its counterpart is the first prespecified public opportunity after the sale becomes public, with an opportunity without an order recorded as a zero; the Laplace flow is the aggregate net demand of the decision interval. In both set-ups the entry ceiling r_C is a tie for the high-cost rival after the top posterior, so the rival's tie rule decides entry at r_C itself.

References